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SAMPLE PAPER TEST – 06 (2020-21)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**
CLASS : XII **DURATION : 3 HRS**

General Instruction:

1. This question paper contains two **parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks
2. **Part-A** has Objective Type Questions and **Part -B** has Descriptive Type Questions
3. Both Part A and Part B have choices.

Part – A:

1. It consists of two sections**- I and II**.
2. Section **I** comprises of 16 very short answer type questions.
3. Section **II** contains **2** case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.

Part – B:

1. It consists of three sections**- III, IV and V.**
2. Section **III** comprises of 10 questions of **2 marks** each**.**
3. Section **IV** comprises of 7 questions of **3 marks** each**.**
4. Section **V** comprises of 3 questions of **5 marks** each**.**
5. Internal choice is provided in **3** questions of Section –III, **2** questions of Section-IV and **3** questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART - A
SECTION-I
Questions 1 to 16 carry 1 mark each.

1. Find the angle between the vectors $\vec{a}$ and $\vec{b}$ if $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 4\hat{i} + 4\hat{j} - 2\hat{k}$.

2. A matrix A of order 3 × 3 has determinant 5. What is the value of | 3A |?

OR

If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$, then find the value of x.

3. Solve the differential equation $\frac{dy}{dx} = 2^{y-x}$

OR

Solve the differential equation $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3}$

4. Check whether the function f: N → N defined by f(x) = 4 – 3x is one-one or not.

5. If a line makes angles 90°, 60° and 30° with the positive directions of x, y and z-axis respectively, then find its direction cosines.

OR

Find the direction cosines of the line passing through two points (2, 1, 0) and (1, –2, 3).

6. Find the area enclosed between the curve $x^2 + y^2 = 16$ and the coordinate axes in the first quadrant.

7. Evaluate: $\int_0^{\pi/2} x\cos x dx$

OR

Evaluate: $\int \cos^3 x \sin x dx$

8. Check whether the relation R = {(1, 1), (2, 2) (3, 3), (1, 2), (2, 3), (1, 3)} on set A = {1, 2, 3} is an equivalence relation or not.

OR

Find the domain of the function $f(x) = \dfrac{1}{\sqrt{\sin x + \sin(\pi + x)}}$ where $\{\cdot\}$ denotes fractional part.

9. Find the vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9.

10. If E and F are events such that 0 < P(F) < 1, then prove that $P(E/F) + P(\overline{E}/F) = 1$

11. Find the value of p for which $p(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

12. Let f: N → N be defined by $f(n) = \begin{cases} \dfrac{n+1}{2}, if \quad n \quad is \quad odd \\ \dfrac{n}{2}, if \quad n \quad is \quad even \end{cases}$ $\forall n \in N$. Find whether the function f is bijective or not.

13. If P(A) = 7/13, P(B) = 9/13 and P(A ∪ B) = 12/13 then evaluate P(A|B).

14. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

15. Simplify: $\tan\theta \begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & -\sec\theta \end{bmatrix} + \sec\theta \begin{bmatrix} -\tan\theta & -\sec\theta \\ -\sec\theta & \tan\theta \end{bmatrix}$

16. Write a 3 × 2 matrix whose elements in the i th row and jth column are given by $a_{ij} = \dfrac{(2i - j)}{2}$.

SECTION-II

Case study-based questions are compulsory. Attempt any four sub parts of each question. Each subpart carries 1 mark

17. In a particular school, Preboard examination is conducted in the month of January 2020 in which, 30% of the students failed in Chemistry, 25% failed in Mathematics and 12% failed in both Chemistry and Mathematics. A student is selected at random.

(i) The probability that the selected student has failed in Chemistry, if it is known that he has failed in Mathematics, is

(a) 3/10 (b) 12/25 (c) 1/4 (d) 3/25

(ii) The probability that the selected student has failed in Mathematics, if it is known that he has failed in Chemistry, is

(a) 22/25 (b) 12/25 (c) 2/5 (d) 3/25

(iii) The probability that the selected student has passed in at least one of the two subjects, is

(a) 22/25 (b) 88/125 (c) 43/100 (d) 3/75

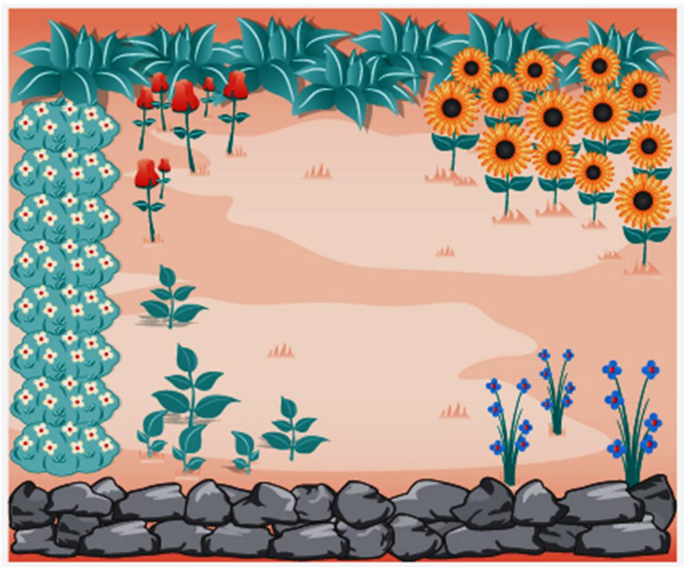
(iv) The probability that the selected student has failed in at least one of the two subjects, is

(a) 2/5 (b) 22/25 (c) 3/5 (d) 43/100

(v) The probability that the selected student has passed in Mathematics, if it is known that he has failed in Chemistry, is

(a) 2/5 (b) 3/5 (c) 1/5 (d) 4/5

18. A farmer has a rectangular garden in his land. He wants to construct fencing using a rock wall on one side of the garden and wire fencing for the other three sides as shown in figure. He has 100 m of wire fencing.



Based on the above information, answer the following questions.

(i) To construct a garden using 100 m of fencing, we need to maximise its

(a) volume (b) area (c) perimeter (d) length of the side

(ii) If x denotes the length of side of garden perpendicular to rock wall and y denote the length of side parallel to rock wall, then find the relation representing total amount of fencing wall.

(a) $x + 2y = 100$ (b) $x + 2y = 50$ (c) $y + 2x = 100$ (d) $y + 2x = 50$

(iii) Area of the garden as a function of x i.e., A(x) can be represented as
(a) $100 + 2x^2$ (b) $x - 2x^2$ (c) $100x - 2x^2$ (d) $100 - x^2$

(iv) Maximum value of A(x) occurs at x equals
(a) 25 m (b) 30 m (c) 26 m (d) 31 m

(v) Maximum area of garden will be
(a) 1200 sq. m (b) 1000 sq. m (c) 1250 sq. m (d) 1500 sq. m

PART B
SECTION – III
Questions 19 to 28 carry 2 marks each.

19. Evaluate: $\int_2^4 \frac{(x^2+x)}{\sqrt{2x+1}}dx$

20. The equation of a line is $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.

OR

Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.

21. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.

22. Prove that: $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$, $x \in [-1/2, 1/2]$

23. An unbiased dice is thrown twice. Let the event A be 'odd number on the first throw' and B be the event 'odd number on the second throw'. Check the independence of the events A and B.

OR

A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white?

24. Determine the value of 'k' for which the following function

$$f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$ is continuous at x = 3.

25. Find the solution of the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2 + 1}{2xy}$ satisfying y(0) = 1.

26. If the matrix A = $\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmteric, then find the value of 'a' and 'b'.

27. Find dy/dx at x = 1, $y = \pi/4$, if $\sin^2 y + \cos xy = k$.

28. Prove that the points A, B and C with position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively are collinear if and only if $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$.

OR

If $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$, then find the projection of $\vec{b}$ on $\vec{a}$.

SECTION – IV

Questions 29 to 35 carry 3 marks each.

29. Show that the relation R in the set of real numbers, defined as R = {(a, b): a $\leq$ b^2} is neither reflexive, nor symmetric, nor transitive.

30. Find the area bounded by the lines y = 1 – ||x| – 1| and the x- axis.

31. Find the equation of tangent to the curve x = sin 3t, y = cos 2t at t = $\pi/4$

OR

Find the intervals in which the function f(x) = $3x^4 - 4x^3 - 12x^2 + 5$ is (a) strictly increasing (b) strictly decreasing

32. Evaluate: $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}}$

33. Let $f(x) = \begin{cases} x + a\sqrt{2}\sin x, 0 \leq x < \frac{\pi}{4} \\ 2x\cot x + b, \frac{\pi}{4} \leq x \leq \frac{\pi}{2} \\ a\cos 2x - b\sin x, \frac{\pi}{2} < x \leq \pi \end{cases}$ be continuous in [0, p], then find the value of a + b.

34. Solve: $(x\sqrt{x^2+y^2} - y^2)dx + xydy = 0$

OR

Solve the differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x$, given that y = 1 when x = $\pi/2$.

35. Show that the function f(x) = |x – 1| + |x + 1|, for all x $\in$ R, is not differentiable at the points x = –1 and x = 1.

SECTION – V

Questions 36 to 38 carry 5 marks each.

36. Find the product BA of matrices $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of linear equations: x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2.

OR

If A = $\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

37. Find the equation of the plane passing through the points (2, 2, –1) and (3, 4, 2) and parallel to the line whose direction ratios are 7, 0, 6.

OR

Find the distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines $\frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}$ and $\frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}$.

38. Solve the following linear programming problem graphically.
Minimize Z = x – 7y + 227 subject to constraints:
$x + y \le 9$; $x \le 7$; $y \le 6$; $x + y \ge 5$; $x, y \ge 0$

OR

Solved the following linear programming problem graphically.
Maximize Z = 11x + 9y subject to constraints:
$180x + 120y \le 1500$; $x + y \le 10$; $x, y \ge 0$